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DIPHTHERIA TRANSMISSION DYNAMICS: A SENSITIVITY ANALYSIS

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Abstract
In this paper, sensitivity analysis of the transmission dynamics of the diphtheria model is considered, using an SIR model. The sensitivity indices of the model parameters were calculated. The study was undertaken to ascertain the parameters that should be considered as targets for intervention strategies. The result shows that an increase in the population of susceptible individuals knowing their infectious status (ρ), the natural death rate (μ), the proportion of aware susceptible persons who have been vaccinated ($\emptyset$), the rate of treatment of diphtheria infectives (δ) and, the disease-induced death rate of infectives (α) will decrease the reproduction number (the response function) thereby curtailing the spread of the disease. while a decrease in these parameters will increase the reproduction number which will result in an escalation of the disease. Similarly, if the recruitment rate (p), the incidence rate of unaware susceptible individuals (β_1), the incidence rate of aware susceptible individuals (β_2), the modification parameter (τ) and the disease-induced death rate of treated diphtheria infectives (v) are increased in value, then the reproduction number will increase and this will lead to an endemic situation. It is recommended that relevant government agencies should closely monitor the sensitivity indices.

Keywords: Diphtheria, Sensitivity Analysis, Reproduction Number, Transmission, Sensitivity Indices.

Introduction

The Centers for Disease Control and Prevention (CDC) (2022) defined diphtheria as "a contagious disease which is caused by strains of bacteria called *Corynebacterium diphtheriae* that make toxin". It is a bacterial infection that affects the mucous membranes of the nose and throat. It is a vaccine-preventable disease that is transmitted through droplets from coughs and sneezes or by intimate contact with a person who has contracted the disease. The disease may impede breathing, and cause heart rhythm problems or death.

In December 2022, the Nigeria Centre for Disease Control (NCDC) disclosed a manifold sudden increase in diphtheria infection affecting different states throughout the country. In June 2023, the NCDC announced the death of a four-year-old resulting from diphtheria infection in the Federal Capital Territory (FCT), Abuja. By 30th June 2023, there were seven hundred and ninety-eight (798) confirmed diphtheria cases from thirty-three (33) local government areas (LGAs) in 8 states, with the FCT inclusive. Seven hundred and eighty-two (782) of the 798 cases occurred in Kano State. Other states with confirmed cases include Lagos, Yobe, Kastina, Cross River, Kaduna and Osun. About 71.7% (0.717) of the 798 infected individuals were children aged 2-14 years. As of 6th July 2023, a case fatality rate (CFR) of 10% (0.10), giving a total of eighty deaths, was reported among all confirmed cases (NCDC, 2023).

The outbreak of the disease continued to pose a serious threat to communities at risk in Nigeria. In July 2023, the number rose from 798 to 1387 confirmed cases, claiming a total of 122 lives, affecting mainly Kano, Yobe, Katsina, Lagos, FCT, Sokoto and Zamfara, accounting for 98% of the suspected cases (UNICEF, 2023 and WHO, 2023). On the 3rd of August 2023, the United Nations International Children's Emergency Fund (UNICEF) reported diphtheria infection of children in 27 states, hence the need to amplify efforts to counter the growing outbreak of the disease.Despite the availability of safe and cost-effective vaccines in the country, it was reported that 654 (that is, 82%) of the 798 initial confirmed diphtheria cases were unvaccinated, including the FCT case. To prevent diphtheria

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infection, CDC (2022) recommends vaccines for infants, children, teens and adults. It was emphasised that early diagnosis and effective treatment are predictors of a favourable outcome.

A crucial objective of mathematical modelling is to find out the most recent result of a system and to find ways to replace any unfavourable result. Varying the values of the most sensitive parameter is the most effective strategy to change the model result (Mikueki, 2012). Sensitivity analysis is crucial for mathematical models as it determines which parameters and initial conditions affect the quantities of the model the most. It investigates changes in a model due to changes in the inputs (initial condition) and reveals which parameters should be given the most attention (Mikueki, 2012). A small variation in a very sensitive parameter will result in a large quantitative change to some quantity of interest which may give rise to qualitatively different results. Hence, in estimating parameters, a sensitive parameter should be carefully estimated. A little change in an insensitive parameter is not likely to give significant changes to any value of interest; hence an insensitive parameter does not require much attention in its estimation.According to Edward et al. (2004), sensitivity analysis determines how sensitive a system is to changes in the structure and parameter values of the model. Since uncertainties are usually associated with parameters, sensitivity analysis helps to build confidence in the model by studying the uncertainties. It is also used to determine how robust a model prediction is to parameter values (Edward, 2004).

Materials and Methods

The total population under consideration at time t, represented by $N(t)$, is split into the following classes of persons susceptible to diphtheria $S(t)$, susceptible individuals who are unaware of diphtheria $S_1(t)$, susceptible persons who know about diphtheria $S_2(t)$, individuals infected with diphtheria $I(t)$ and recovered persons $R(t)$. Hence,

$$N(t) = S(t) + S_1(t) + S_2(t) + I(t) + R(t). \tag{1}$$

In this work, the model proposed by Udoo (2018) is adopted and is stated as follows:

$$\frac{dS}{dt} = p - (\lambda + \rho + \mu)S, \qquad S(0) > 0 \tag{2}$$

$$\frac{dS_1}{dt} = (1 - \phi)\rho S - (\tau\lambda_1 + \mu)S_1, \qquad S_1(0) > 0 \tag{3}$$

$$\frac{dS_2}{dt} = \phi\rho S - \mu S_2, \qquad S_2(0) > 0 \tag{4}$$

$$\frac{dI}{dt} = \lambda S + \tau\lambda_1 S_1 - (\delta + \alpha + \mu)I, \qquad I(0) \geq 0 \tag{5}$$

$$\frac{dR}{dt} = \delta I - (v + \mu)R, \qquad R(0) \geq 0 \tag{6}$$

The variables and parameter values in the model are presented in Tables 1-2 below.

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Table 1: Parameters in the system (2) – (6)

Parameter	Description	Approximate value
p	Recruitment rate	0.029
β_1	Incidence rate of unaware susceptible individuals	$0 \leq \beta_1 \leq 1$
β_2	Incidence rate of aware susceptible individuals	$0 \leq \beta_2 \leq 1$
τ	Modification parameter	$0 < \tau < 1$
ρ	Rate of susceptible knowing their infectious status	$0 < \rho < 1$
μ	Natural death rate	0.02
$\emptyset$	The proportion of aware susceptible individuals who are vaccinated	$0 < \emptyset < 1$
δ	Treatment rate of diphtheria infectives	$0 < \delta < 1$
α	Disease-induced death rate of infectives	0.10
v	Disease-induced death rate of treated diphtheria infectives	0.02

Table 2: Description of model variables

Parameter	Description
S	Population of individuals susceptible to diphtheria
S_1	The population of susceptible persons not knowing diphtheria
S_2	Population of susceptible persons knowing diphtheria
I	The population of Infectious Individuals
R	Population of recovered individuals

It follows from (1) that the rate at which the total population is changing is given by

$$\frac{dN(t)}{dt} = p - \mu N - \alpha I - vR \quad (7)$$

The forces of infection for S and S_1 are given as

$\lambda = \beta_1 I$ and $\lambda_1 = \beta_2 I$

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Basic Properties of the Model
Since equations (2) – (6) monitor the human population, it is assumed that all the state variables and parameters are non-negative for all time (t). In other words, the solution of the model equations (2) – (6) with positive initial data will remain positive for all $t \geq 0$.

Existence and Uniqueness of Solution To establish the conditions for the existence and uniqueness of the solution for the model (2) – (6), let

$$f_1(t,x) = p - (\lambda + \rho + \mu)S, \tag{8}$$

$$f_2(t,x) = (1-\phi)\rho S - (\tau\lambda_1 + \mu)S_1, \tag{9}$$

$$f_3(t,x) = \phi\rho S - \mu S_2, \tag{10}$$

$$f_4(t,x) = \lambda S + \tau\lambda_1 S_1 - (\delta + \alpha + \mu)I, \tag{11}$$

$$f_5(t,x) = \delta I - (v + \mu)R. \tag{12}$$

So that

$$\frac{dx}{dt} = f(t,x) = f(x). \tag{13}$$

Theorem 1. Let D' represent the region

$$|t' - t'_0| \leq a,\ \|x' - x'_0\| \leq b,\ x = (x'_1, x'_2, \dots, x'_n) = (x'_{10}, x'_{20,} \dots, x'_{n0}) \tag{14}$$

andassume that $f(t', x')$ meets the Lipschitz condition

$$\|f(t', x'_1) - f(t'_1, x'_2)\| \leq k\|x'_1 - x'_2\| \tag{15}$$

for(t', x'_1) and (t'_1, x'_2) inD' and$k > 0$. Then, there is a constant $\delta > 0$ such that there is a unique continuous vector solution $\bar{x}'(t)$ of equations (8) – (12) in $|t' - t'_0| \leq \delta$.

$\frac{\partial f_i}{\partial x'_j},\ i,j = 1,2,\dots,n$ is continuous and bounded in D' and met the condition in equation (15)

Lemma 1. If $f(t', x')$ is continuous and has partial derivative $\frac{\partial f_i}{\partial x'_j}$ on a bounded closed convex domain $\mathbb{R}$, then it satisfies a Lipschitz condition in $\mathbb{R}$.

The region of interest is given by

$$1 \leq \epsilon \leq \mathbb{R} \tag{16}$$

and bounded solution of the form below is sought for:

$$0 < \mathbb{R} < \infty \tag{17}$$

The following existence theorem will be proved:

Theorem 2: If D' represents the region defined in (15) such that (16) and (17) are true, then $\forall$ a solution of model (8) – (12) is bounded in the region D'.

Proof. Let

$$f_1(t', x') = p - (\lambda + \rho + \mu)S,$$

$$f_2(t', x') = (1 - \phi)\rho S - (\tau\lambda_1 + \mu)S_1,$$

$$f_3(t', x') = \phi\rho S - \mu S_2,$$

$$f_4(t', x') = \lambda S + \tau\lambda_1 S_1 - (\delta + \alpha + \mu)I,$$

$$f_5(t', x') = \delta I - (v + \mu)R.$$

It is sufficient to prove that the continuity of $\frac{\partial f_i}{\partial x'_j}, i = j = 1, 2, 3, 4, 5$ exist. Differentiating f_i partially with respect to the state variables S, S_1, S_2, I and R, yield:

$$\frac{\partial f_1}{\partial S} = -(\lambda + \rho + \mu), \quad \left|\frac{\partial f_1}{\partial S}\right| = |-(\lambda + \rho + \mu)| < \infty \tag{18}$$

$$\frac{\partial f_1}{\partial S_1} = 0, \qquad \left|\frac{\partial f_1}{\partial S_1}\right| = |0| < \infty \tag{19}$$

$$\frac{\partial f_1}{\partial S_2} = 0, \qquad \left|\frac{\partial f_1}{\partial S_2}\right| = |0| < \infty \tag{20}$$

$$\frac{\partial f_1}{\partial I} = 0, \qquad \left|\frac{\partial f_1}{\partial I}\right| = |0| < \infty \tag{21}$$

$$\frac{\partial f_1}{\partial R} = 0, \qquad \left|\frac{\partial f_1}{\partial R}\right| = |0| < \infty \tag{22}$$

Also,

$$\frac{\partial f_2}{\partial S} = (1 - \phi)\rho, \qquad \left|\frac{\partial f_2}{\partial S}\right| = |(1 - \phi)\rho| < \infty \tag{23}$$

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$$\frac{\partial f_2}{\partial S_1} = -(\tau\lambda_1 + \mu), \quad \left|\frac{\partial f_2}{\partial S_1}\right| = |-(\tau\lambda_1 + \mu)| < \infty \tag{24}$$

$$\frac{\partial f_2}{\partial S_2} = 0, \quad \left|\frac{\partial f_2}{\partial S_2}\right| = |0| < \infty \tag{25}$$

$$\frac{\partial f_2}{\partial I} = 0, \quad \left|\frac{\partial f_2}{\partial I}\right| = |0| < \infty \tag{26}$$

$$\frac{\partial f_2}{\partial R} = 0, \quad \left|\frac{\partial f_2}{\partial R}\right| = |0| < \infty \tag{27}$$

Similarly,

$$\frac{\partial f_3}{\partial S} = \phi\rho, \quad \left|\frac{\partial f_3}{\partial S}\right| = |\phi\rho| < \infty \tag{28}$$

$$\frac{\partial f_3}{\partial S_1} = 0, \quad \left|\frac{\partial f_3}{\partial S_1}\right| = |0| < \infty \tag{29}$$

$$\frac{\partial f_3}{\partial S_2} = -\mu, \quad \left|\frac{\partial f_3}{\partial S_2}\right| = |-\mu| < \infty \tag{30}$$

$$\frac{\partial f_3}{\partial I} = 0, \quad \left|\frac{\partial f_3}{\partial I}\right| = |0| < \infty \tag{31}$$

$$\frac{\partial f_3}{\partial R} = 0, \quad \left|\frac{\partial f_3}{\partial R}\right| = |0| < \infty \tag{32}$$

Furthermore,

$$\frac{\partial f_4}{\partial S} = \lambda, \quad \left|\frac{\partial f_4}{\partial S}\right| = |\lambda| < \infty \tag{33}$$

$$\frac{\partial f_4}{\partial S_1} = \tau\lambda_1, \quad \left|\frac{\partial f_4}{\partial S_1}\right| = |\tau\lambda_1| < \infty \tag{34}$$

$$\frac{\partial f_4}{\partial S_2} = 0, \quad \left|\frac{\partial f_4}{\partial S_2}\right| = |0| < \infty \tag{35}$$

$$\frac{\partial f_4}{\partial I} = -(\delta + \alpha + \mu), \quad \left|\frac{\partial f_4}{\partial I}\right| = |-(\delta + \alpha + \mu)| < \infty \tag{36}$$

$$\frac{\partial f_4}{\partial R} = 0, \quad \left|\frac{\partial f_4}{\partial R}\right| = |0| < \infty \tag{37}$$

Finally,

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$$\frac{\partial f_5}{\partial S} = 0, \qquad \left|\frac{\partial f_5}{\partial S}\right| = |0| < \infty \tag{38}$$

$$\frac{\partial f_5}{\partial S_1} = 0, \qquad \left|\frac{\partial f_5}{\partial S_1}\right| = |0| < \infty \tag{39}$$

$$\frac{\partial f_5}{\partial S_2} = 0, \qquad \left|\frac{\partial f_5}{\partial S_2}\right| = |0| < \infty \tag{40}$$

$$\frac{\partial f_5}{\partial I} = \delta, \qquad \left|\frac{\partial f_5}{\partial I}\right| = |\delta| < \infty \tag{41}$$

$$\frac{\partial f_5}{\partial R} = -(v+\mu), \qquad \left|\frac{\partial f_5}{\partial R}\right| = |-(v+\mu)| < \infty \tag{42}$$

It has been shown that the partial derivatives (18) – (42) of the right-hand side of (2) – (6) with respect to S, S_1, S_2, I, R are continuously differentiable and bounded. Hence, by Theorem 2, it is locally Lipschitz, therefore, $S(t), S_1(t), S_2(t), I(t), R(t)$ is a unique solution to system (2) – (6) with the initial condition $S_0, S_{10}, S_{20}, I_0, R_0$ in the region D'.

Invariant Region

Lemma 2. The region $D \subset \mathbb{R}^5_+$ is positively invariant for the equation (2) – (6) with zero or positive initial condition in $\mathbb{R}^5_+$.

Proof. From Equation (7), it is shown that,

$$\frac{dN}{dt} = p - \mu N - \alpha I - vR$$

$$\Rightarrow \frac{dN}{dt} \leq p - \mu N$$

$$\Rightarrow N(t) \leq N(0)e^{\mu t} + \frac{p}{\mu}(1 - e^{-\mu t})$$

If $N(0) \leq 0$, then $N(t) \leq \frac{p}{\mu}$. Hence, equations (2) – (6) will be studied in the feasible region $D \subset \mathbb{R}^5_+$, with

$$D = \left\{S,\ S_1,\ S_2,\ I,\ R \in \mathbb{R}^5_+ :\ 0 \leq N \leq \frac{p}{\mu}\right\}.$$

Thus, D is a positively invariant set and a global attractor of the system (2) – (6). That is, any phase trajectory initiated anywhere in the non-negative region $\mathbb{R}^5_+$ of the phase space eventually enters region D and remains in D thereafter.

Positivity and Boundedness of Solutions

Lemma 3. The solution of the system (2) – (6), $\{S,\ S_1,\ S_2,\ I,\ R\}$, with initial condition, $\{S_0,\ S_{10},\ S_{20},\ I_0,\ R_0 \geq 0\} \in D$, will remain greater than zero for all time $t \geq 0$.

Proof. From equation (2),

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$$\frac{dS}{dt} = p - \lambda S - \rho S - \mu S$$

$$\geq -\lambda S - \rho S - \mu S$$

$$\int \frac{dS}{S} = -\int (\lambda + \rho + \mu)\, dt$$

$$\Rightarrow S \geq S_0 e^{-\int (\lambda+\rho+\mu) dt} \geq 0$$

Similarly, it can be shown that $S_1(t) \geq 0$, $S_2 \geq 0$, $I \geq 0$, $R \geq 0$ for all time $t > 0$.

Critical Points and Basic Reproduction Number (R_0)

Equations (2) – (6) have two critical points: Disease-free equilibrium (DFE) and endemic equilibrium (EE) points. At equilibrium, the right-hand side of the model (2) – (6) is zero (George, 2019). Hence

$$(S^*,\ S_1^*,\ S_2^*,\ I^*,\ R^*) = \left(\frac{p}{\rho+\mu}, \frac{(1-\emptyset)\rho p}{\mu(\rho+\mu)}, \frac{\emptyset \rho p}{\mu(\rho+\mu)}, 0, 0\right) \quad (43)$$

The basic reproduction number (R_0) is the average number of secondary infections when a typical infective enters a susceptible population. In this study, R_0 is the average number of new diphtheria infections generated by a single diphtheria-infected individual throughout infection of the individual in a completely susceptible population (Diekmann et al., 1990; Anderson & May, 1991; Hethcote, 2000; Driessche&Watmough, 2002).The magnitude of R_0 not only indicates the speed of how a disease will spread, but whether it will spread at all.
Using the next-generation matrix, the basic reproduction number (R_0) can be determined. According to Driessche and Watmough (2002), the basic reproduction number (R_0) is given by the dominant eigenvalue of FV^{-1}, where$F$ and V respectively represent the new infection term and the remaining transfer terms Hence,

$F = \lambda S + \tau\lambda_1 S_1$ and $V = \delta + \alpha + \mu$. So that, $V^{-1} = \frac{1}{\delta+\alpha+\mu}$.

$$\therefore R_0 = FV^{-1}$$

$$= \frac{\lambda S^* + \tau\lambda_1 S_1^*}{(\delta+\alpha+\mu)} = \frac{\beta_1 p}{(\rho+\mu)(\delta+\alpha+\mu)} + \frac{\tau\beta_2(1-\phi)\rho p}{\mu(\rho+\mu)(\delta+\alpha+\mu)} = \frac{\boldsymbol{p}[\mu\beta_1 + \tau\beta_2(1-\phi)\rho]}{(\rho+\mu)(\delta+\alpha+\mu)} \quad (44)$$

Sensitivity Analysis
The sensitivity indices of the parameters in the model with respect to R_0 are computed as follows:

$$p: \quad X_p^{R_0} = \frac{\partial R_0}{\partial p} \times \frac{p}{R_0} = \left[\frac{[\mu\beta_1 + \tau\beta_2(1-\phi)\rho]}{\mu(\rho+\mu)(\delta+\alpha+\mu)}\right] \times \frac{p\mu(\rho+\mu)(\delta+\alpha+\mu)}{\boldsymbol{p}[\mu\beta_1 + \tau\beta_2(1-\phi)\rho]} = 1$$

$$\beta_1: \quad X_{\beta_1}^{R_0} = \frac{\partial R_0}{\partial \beta_1} \times \frac{\beta_1}{R_0} = \left[\frac{p\mu}{\mu(\rho+\mu)(\delta+\alpha+\mu)}\right] \times \frac{\beta_1\mu(\rho+\mu)(\delta+\alpha+\mu)}{\boldsymbol{p}[\mu\beta_1 + \tau\beta_2(1-\phi)\rho]}$$

$$= \frac{\mu\beta_1}{\mu\beta_1 + \tau\beta_2(1-\phi)\rho} = \frac{0.02\times0.8}{0.02\times0.8 + 0.6\times0.7(1-0.5)0.6} = \frac{0.016}{0.142} = 0.1127$$

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$$\beta_2: \quad X_{\beta_2}^{R_0} = \frac{\partial R_0}{\partial \beta_2} \times \frac{\beta_2}{R_0} = \left[\frac{p\tau(1-\phi)\rho}{\mu(\rho+\mu)(\delta+\alpha+\mu)}\right] \times \frac{\beta_2\mu(\rho+\mu)(\delta+\alpha+\mu)}{p[\mu\beta_1+\tau\beta_2(1-\phi)\rho]}$$

$$= \frac{\beta_2\tau(1-\phi)\rho}{\mu\beta_1+\tau\beta_2(1-\phi)\rho} = \frac{0.7\times0.6\times0.5\times0.6}{0.02\times0.8+0.6\times0.7(1-0.5)0.6} = \frac{0.126}{0.142} = 0.8873$$

$$\tau: \quad X_{\tau}^{R_0} = \frac{\partial R_0}{\partial \tau} \times \frac{\tau}{R_0} = \left[\frac{p\beta_2(1-\phi)\rho}{\mu(\rho+\mu)(\delta+\alpha+\mu)}\right] \times \frac{\tau\,\mu(\rho+\mu)(\delta+\alpha+\mu)}{p[\mu\beta_1+\tau\beta_2(1-\phi)\rho]}$$

$$= \frac{\beta_2\tau(1-\phi)\rho}{\mu\beta_1+\tau\beta_2(1-\phi)\rho} = \frac{0.7\times0.6\times0.5\times0.6}{0.02\times0.8+0.6\times0.7(1-0.5)0.6} = \frac{0.126}{0.142} = 0.8873$$

$$\rho: \quad X_{\rho}^{R_0} = \frac{\partial R_0}{\partial \rho} \times \frac{\rho}{R_0} = \frac{p\{\tau\beta_2(\rho+\mu)(1-\phi)-[\mu\beta_1+\tau\beta_2(1-\phi)\rho]\}}{\mu(\rho+\mu)^2(\delta+\alpha+\mu)} \times \frac{\rho\,\mu(\rho+\mu)(\delta+\alpha+\mu)}{p[\mu\beta_1+\tau\beta_2(1-\phi)\rho]}$$

$$= \frac{\rho\{\tau\beta_2(\rho+\mu)(1-\phi)-[\mu\beta_1+\tau\beta_2(1-\phi)\rho]\}}{\mu(\rho+\mu)[\mu\beta_1+\tau\beta_2(1-\phi)\rho]}$$

$$= \frac{0.6\{0.6\times0.7(0.6+0.02)(1-0.5)-[0.02\times0.8+0.6\times0.7(1-0.5)0.6]\}}{0.02(0.6+0.02)[0.02\times0.8+0.6\times0.7(1-0.5)0.6]}$$

$$= \frac{-0.00708}{0.1544} = -0.0459$$

$$\mu: \quad X_{\mu}^{R_0} = \frac{\partial R_0}{\partial \mu} \times \frac{\mu}{R_0} = \frac{\mu(\rho+\mu)(\delta+\alpha+\mu)\beta_1-[\mu\beta_1+\tau\beta_2(1-\phi)\rho][(\rho+2\mu)(\delta+\alpha+\mu)+\mu(\rho+\mu]}{(\rho+\mu)(\delta+\alpha+\mu)[\mu\beta_1+\tau\beta_2(1-\phi)\rho]}$$

$$= \frac{0.02(0.62)(0.7)(0.8)-[(0.02)(0.8)+(0.6)(0.7)(0.5)(0.6)](0.64)(0.7)+(0.02)(0.62)}{(0.6+0.02)(0.5+0.10+0.02)[0.02\times0.8+0.6\times0.7(1-0.5)0.6]}$$

$$= -\frac{0.3567}{0.0521} = -6.8464$$

$$\emptyset: \quad X_{\emptyset}^{R_0} = \frac{\partial R_0}{\partial \emptyset} \times \frac{\emptyset}{R_0} = \left[-\frac{p\tau\beta_2\rho}{\mu(\rho+\mu)(\delta+\alpha+\mu)}\right] \times \frac{\emptyset\mu(\rho+\mu)(\delta+\alpha+\mu)}{p[\mu\beta_1+\tau\beta_2(1-\phi)\rho]}$$

$$= \frac{\beta_2\tau\emptyset\rho}{\mu\beta_1+\tau\beta_2(1-\phi)\rho} = \frac{0.7\times0.6\times0.5\times0.6}{0.02\times0.8+0.6\times0.7(1-0.5)0.6} = \frac{0.126}{0.142} = -0.8873$$

$$\alpha: \quad X_{\alpha}^{R_0} = \frac{\partial R_0}{\partial \alpha} \times \frac{\alpha}{R_0} = \frac{-p[\mu\beta_1+\tau\beta_2(1-\phi)\rho]\mu(\rho+\mu)}{\mu^2[(\rho+\mu)(\delta+\alpha+\mu)]^2} \times \frac{\alpha\mu(\rho+\mu)(\delta+\alpha+\mu)}{p[\mu\beta_1+\tau\beta_2(1-\phi)\rho]}$$

$$= -\frac{(\rho+\mu)\alpha}{\mu(\rho+\mu)(\delta+\alpha+\mu)}$$

$$= -\frac{(0.6+0.02)0.10}{0.02(0.6+0.02)(0.5+0.10+0.02)} = -\frac{0.062}{0.007688} = -8.0645$$

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δ $\quad X_{\delta}^{R_0} = \frac{\partial R_0}{\partial \delta} \times \frac{\delta}{R_0} = \frac{-p[\mu\beta_1+\tau\beta_2(1-\phi)\rho]\mu(\rho+\mu)}{\mu^2[(\rho+\mu)(\delta+\alpha+\mu)]^2} \times \frac{\delta\mu(\rho+\mu)(\delta+\alpha+\mu)}{p[\mu\beta_1+\tau\beta_2(1-\phi)\rho]}$

$$= -\frac{(\rho+\mu)\delta}{\mu(\rho+\mu)(\delta+\alpha+\mu)}$$

$$= -\frac{(0.6+0.02)0.5}{0.02(0.6+0.02)(0.5+0.10+0.02)} = -\frac{0.31}{0.007688} = -40.3226$$

v: $\quad X_{v}^{R_0} = \frac{\partial R_0}{\partial v} \times \frac{v}{R_0} = \frac{v\mu(\rho+\mu)(\delta+\alpha+\mu)}{p[\mu\beta_1+\tau\beta_2(1-\phi)\rho]}$

$$= \frac{(0.02)(0.02)(0.6+0.02)(0.5+0.10+0.02)}{[0.02\times 0.8+0.6\times 0.7(1-0.5)0.6]0.029} = \frac{.00015376}{0.004118} = 0.0373$$

A summary of the sensitivity indices calculated above is given in Table 3 below:

Table 3: Numerical values of the sensitivity indices

Parameter	Sensitivity index
p	1.0000
β_1	0.1127
β_2	0.8873
τ	0.8873
ρ	−0.0459
μ	−6.8464
$\emptyset$	−0.8873
δ	−40.3226
α	−8.0645
v	0.0373

A positive index indicates that, as the value increases, the reproduction number (R_0) increases and an increase in the negative index will decrease the value of R_0.

Effect of the Parameters on the Reproduction Number (R_0)

The rate of susceptible individuals knowing their infectious status (ρ), the natural death rate (μ), the proportion of aware susceptible individuals who are vaccinated ($\emptyset$), the treatment rate of diphtheria infectives (δ) and disease-induced death rate of infectives (α) are the negatively indexed parameters. Increasing these parameters will decrease the reproduction number (the response function) thereby containing the transmission of the virus. On the other hand, recruitment rate (p), incidence rate of unaware susceptible individuals (β_1), incidence rate of aware susceptible individuals (β_2), modification parameter (τ) and disease-induced death rate of treated diphtheria infectives (v) are the positively indexed parameters. Increasing these parameters will increase the value of the reproduction number and the disease will be endemic.

Conclusion

Since the parameters with negative indices are the most significant parameters affecting the reproduction number, these parameters must be the target for control intervention strategies. An increase in the values of these parameters will decrease the reproduction number, while a decrease in the parameters will increase the reproduction number. If

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the reproduction number increases, then there will be an escalation of the disease. Similarly, if the positively indexed parameters increase in value, then the reproduction number will increase and this will lead to the prevalence of the disease. It is therefore recommended that relevant agencies closely monitor the sensitivity indices.

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